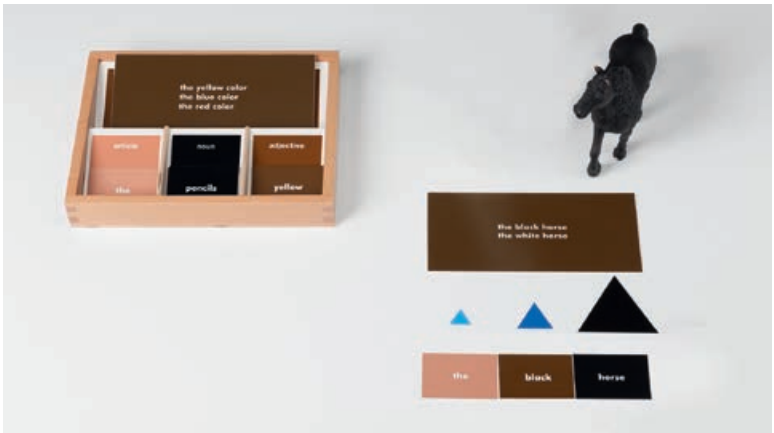
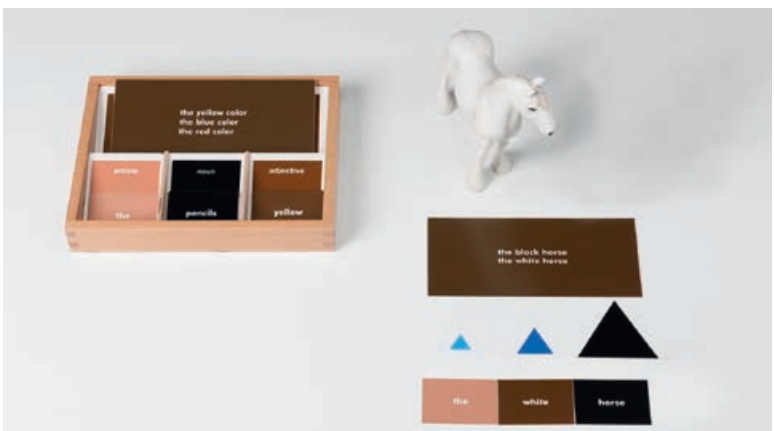


Box III A: the black horse



Box III A: the white horse

Box III: Adjectives
B: sense of sight: size

the broad shelf the narrow shelf
the tall child the short child
the large cup the small cup
the long ruler the short ruler
the wide strip the narrow strip
the sharp needle the blunt needle
the thick fabric the thin fabric

Box III: Adjectives
C: sense of sight: shape

the acute-angled triangle the obtuse-angled triangle the right-angled triangle
the quadrangular pyramid the triangular pyramid
the triangular inset the square inset the circular inset the rectangular inset
the cylindrical box the prismatic box
the equilateral triangle the isosceles triangle the scalene triangle

especially adapted to the child's home activities. Using the beads clarifies the different steps of the operation, creating almost *a rational arithmetic* which supersedes the common empirical methods, that reduce the mechanism of abstract operations to a simple *routine.* For this reason, these pastimes prepare the way for the rational processes of mathematics which the child meets in the higher grades.

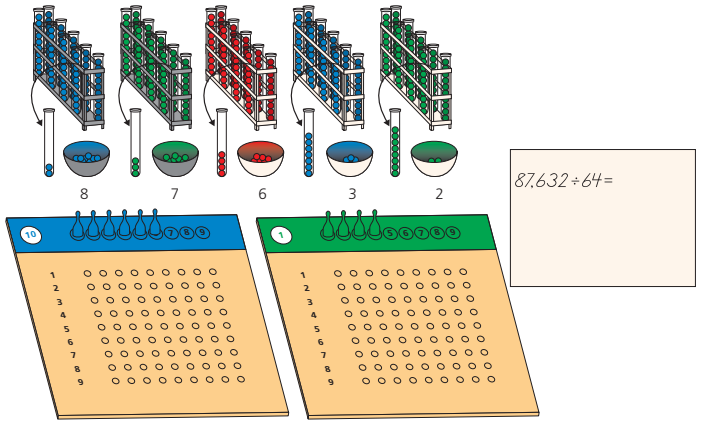
The bead frame will no longer suffice here. We need the unit division board. However, we require several of such boards and an adequate provision of beads. The work is too complicated to be described clearly, but in practise it is easy and most interesting.

It is sufficient here to suggest the method of procedure with the material. The units, tens, hundreds, etc., are expressed by different-coloured beads: *units,* green; *tens,* blue; *hundreds,* red. Then there are racks of different colours: *white* for the simple units, tens, and hundreds; *grey* for the thousands; *black* for the millions. There also is a tray in which the racks are placed. Each rack contains ten tubes with ten beads in each.

There are cups: Three cups are white outside and green, blue and red inside. The next three are grey outside and green, blue and red inside. The last cup is black outside and green inside.

Suppose we must divide 87,632 by 64. Five of the racks are put in a row, arranged from left to right according to the value of their colour, as follows: two grey cups – one blue inside and the other green – and three white cups with the inside respectively red, blue, and green.

In the first cup to the left we put 8 blue beads; in the second cup 7 green beads; in the third, 6 red beads; in the fourth 3 blue beads; and in the fifth box 2 green beads.

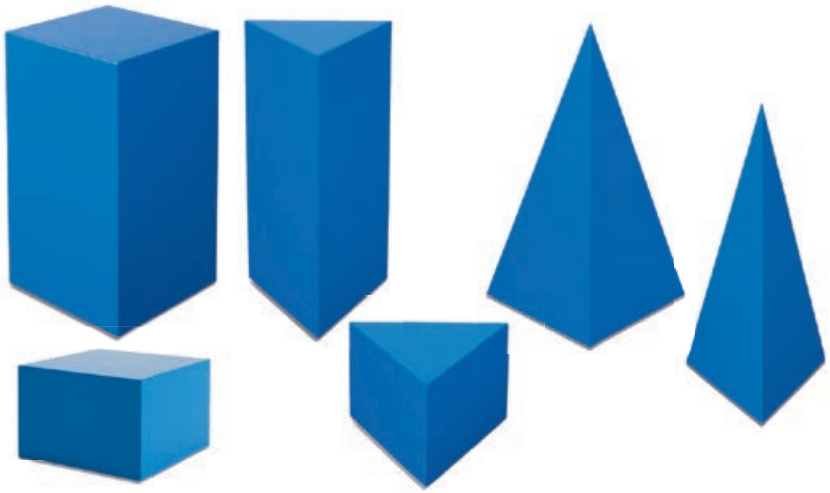


Behind each cup is one of the racks with ten tubes filled with beads of corresponding colours. These beads – ten in each tube – are used in exchanging the units of a higher denomination for those of a lower.

There are two division boards, one next to the other, placed below the row of cups. In the one to the left, with a blue edge and holes for blue skittles, 6 blue skittles are placed. In the one to the right with a green edge and holes for green skittles, 4 green skittles are placed.

Now to divide 87,632 by 64, place the first two cups at the left (containing 8 and 7 beads respectively) above the two division boards. On the first board the eight beads are arranged in two rows of six, as in the more simple division. On the second board the seven beads are arranged in one rows of four, corresponding to the number indicated by the green skittles, and three on the next row.

The two quotients must be reduced with reference to the quotient in the first division board. All the other is considered as a remainder. The



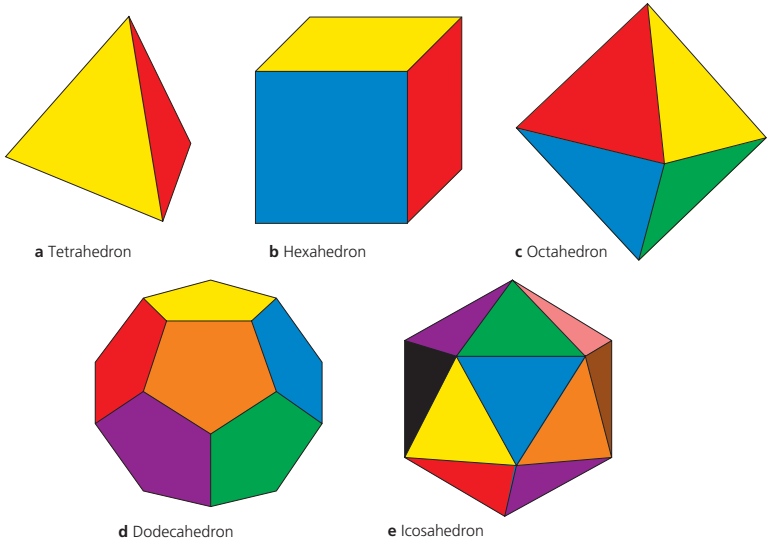
Having mastered these fundamental ideas, it is easy to study the rest, and few explanations will be needed.

In many cases the incentive to do original problems may be developed by giving the children definite examples: as, how can the area of a circle be found? the volume of a cylinder? Of a cone? Problems on the total area of some solids also may be suggested. Many times the children will risk spontaneous inductions and often of their own accord proceed to measure the total surface area of all the solids at their disposal, even going back to the materials used in the "Children's House".

The material includes a series of wooden solids with a base measurement of 10 cm.:

A quadrangular parallelopiped (10 x 10 x 20 cm.)
A quadrangular parallelopiped equal to $\frac{1}{3}$ of above
A quadrangular pyramid (10 x 10 x 20 cm.)
A triangular prism (10 x 20 cm.)
A triangular prism equal to $\frac{1}{3}$ of above
The corresponding pyramid (10 x 20 cm.)
A cylinder (10 cm. diameter, 20 altitude)
A cylinder equal to $\frac{1}{3}$ of above
A cone (10 cm. diameter, 20 altitude)
A sphere (10 cm. diameter)

An ovoid (maximum diameter 10 cm.)
An ellipsoid (maximum diameter 10 cm.)
Regular Polyhedrons
Tetrahedron
Hexahedron (cube)
Octahedron
Dodecahedron
Icosahedron
(The faces of these polyhedrons are in different colours.)

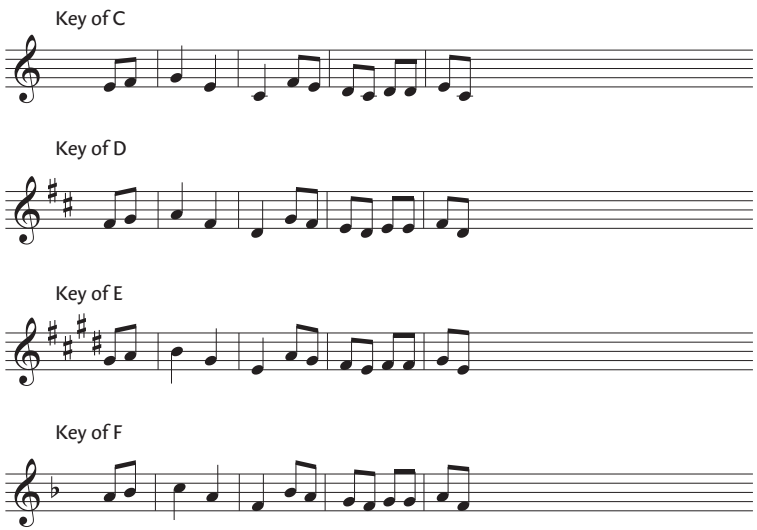


Since in using the first material, by changing the third and sixth bell, the child was taught to recognize the harmonic minor scale, to construct it and listen to it, it is now an obviously simple matter for him to make up all the minor scales.

Here is a specimen of key transposition from major to minor:

1		2		3	4		5		6		7	8
1		2	3		4		5	6			7	8

We have thus developed exercises, which prepare for the recognition of the major and minor tones. It also becomes an easy matter to play a simple *motif* in different keys. It is sufficient to move the pattern strip, as has been indicated, and play them over according to the indications of the numbers of the pattern strip.



At this point children usually develop great keenness for producing sounds and scales on all kinds of instruments (stringed instruments, wind instruments, etc.).

One of the instruments which brings the child to producing and recognizing notes is the *monochord*. It is a simple, resonant box with one string. The first exercise is in tuning. The string is made to correspond with one of the resonant prisms (c). This is made possible by a key with which the string can be loosened or tightened. The child may now be taught how to handle the violin bow or mandolin plectrum, or may be instructed in the finger thrumming used for the harp or banjo.

On one of our monochords, the notes are indicated by fixed transversal frets, the name of each note being printed in the proper space. These notes are, however, not written on the other monochord, where the child must learn to discover by ear the proper distances at which the notes are produced. In this case the child has at his disposal movable frets with which he can indicate the points he has discovered as producing a given note. These frets should be left in position by the child to serve as a check on his work. The children have shown considerable interest also in little pitch pipes, which give very pleasing tones.

The monochord. In the first instrument the notes are indicated by frets. On the monochord in the foreground the child places the frets as he discovers the notes by drawing the bow across the string.